DOCUMENT RESUME

ED 048 362 TM 000 440

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TITLE	A Theorem to Confirm Causal Directions in a Closed System of Five Variables.
PUB DATE	Oct 70
NOTE	14p.; Paper presented at the Annual Meeting of the American Educational Research Association, New York, New York, February 1971
EDRS PRICE	EDRS Price MF-$0.65 HC-$3.29
DESCRIPTORS	*Correlation, Educational Research, *Mathematical Models, *Multiple Regression Analysis, *Research Methodology, *Statistical Analysis, Systems Analysis
IDENTIFIERS	*Causal Theory, Monte Carlo Methods

ABSTRACT

The applicability of a mathematical theorem designed to trace causality of a three-variable path that consists of an initial cause variable, an intermediate variable, and a final-effect variable with control over other system variables is evaluated. The formula was used with a horizontal rather than a normal distribution, as had been done in an earlier study which successfully demonstrated the theorem. Horizontally-distributed random numbers (200x5) were generated and substituted into five interdependent equations to produce five scores for each of 200 pseudo people. Methods and techniques, data sources, and results are presented. Results indicate that the model is reliable, potentially useful to the practitioner as a multivariate tool in educational research, and a possible instrument for the investigation of causality. (AE)

A THEOREM

TO CONFIRM CAUSAL DIRECTIONS

IN A CLOSED SYSTEM OF FIVE VARIABLES

By

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October 1970

Presented in Session D18: Applications of Monte Carlo Methods
American Educational Research Association Annual Meeting
Paper Session: Synopsis, Division D
New York City, New York
February 5, 1971

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ABSTRACT

A theorem derived from a previous study was tested in this study.

THEOREM: In a 3-variable path consisting of initial-cause variable A, intermediate variable B, and final-effect variable C, with control for other system variables K:

$$|d\beta_{cb\cdot k/a}| < |d\beta_{ab\cdot k/c}|$$

where a and c are stepped into regression to obtain beta differentials,

$$|\beta_{ca\cdot kb}| < |\beta_{ac\cdot kb}|,$$

$$|r_{ac\cdot k}| < |r_{ab\cdot k}| < |r_{bc\cdot k}|,$$

and

$$|r_{ac\cdot kb}| < |r_{ab\cdot kc}| < |r_{bc\cdot ka}|.$$

Duplicating the previous study but with the important difference that horizontal rather than normal distributions were used, the present study yielded the finding that the theorem was reliable and potentially useful.

A THEOREM TO CONFIRM CAUSAL DIRECTIONS

IN A CLOSED SYSTEM OF FIVE VARIABLES

George A. Nigro

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A paper presented at the 1969 AERA Annual Meeting[1] exposed a set of inequalities based on the development of a "beta index." This index was proposed as a confirming device for causal directions of three-variable paths in a stepped-regression and correlation analysis from dummy data. These inequalities were completely consistent for causal directions but were inconsistent for acausal directions. The set was generalized to four mathematical statements of asymmetry and proposed as a theorem for this paper.

THEOREM: In a three-variable path consisting of initial-cause variable A, intermediate variable B, and final-effect variable C, with control for other system variables K

$$|d\beta_{cb\cdot k/a}| < |d\beta_{ab\cdot k/c}| \tag{1}$$

where a and c are stepped into regression to obtain beta differentials,

$$|\beta_{ca\cdot kb}| < |\beta_{ac\cdot kb}|, \tag{2}$$

$$|r_{ac\cdot k}| < |r_{ab\cdot k}| < |r_{bc\cdot k}|, \tag{3}$$

and

$$|r_{ac\cdot kb}| < |r_{ab\cdot kc}| < |r_{bc\cdot ka}|. \tag{4}$$

The purpose of the present paper is to report on the study to test the theorem in a duplication of the previous study but with one important difference that horizontal rather than normal distributions were used. Since the theorem could be demonstrated with normal distributions, would dispersions extending the input data to the limits of horizontal distributions upset the theorem?

One might argue intuitively that the theorem should be robust if it is to be accepted for wide usage. But it should be rigorously tested first. If then also demonstrated as tenable, speculation should be then made as to the application of the theorem with real data.

METHODS AND TECHNIQUES

200x5 horizontally-distributed random numbers, R_i, were generated and substituted into five interdependent equations to produce five scores, X_i, for each of 200 pseudo-people, as follows.

$$X_1 = R_1 \tag{5}$$

$$X_2 = R_2 \tag{6}$$

$$X_3 = b_{31}X_1 + b_{32}X_2 + R_3 \tag{7}$$

$$X_4 = b_{41}X_1 + b_{42}X_2 + R_4 \tag{8}$$

The data were then computerized and subjected to stepped-regression and correlation analysis. Beta and correlation coefficients produced which resulted follow the usual explanations given in texts, whereas the beta differentials require a set procedure outlined in the previous paper[1]. Basically, for any given beta weight for variable A on the intermediate variable B, with control for other system variables K, the last variable C is stepped into regression to produce a new beta weight on variable B; the algebraic difference between the two beta weights is found and its absolute value is the "beta differential" for variable A on variable B, $d\beta_{ab \cdot k/c}$ or simply $d\beta_{ab}$, as given in the right-hand member of Inequality 1. In like manner, the beta differential for variable C on variable B is found for comparison. Comparison of the beta differentials should demonstrate or fail to demonstrate the <u>beta constancy principle</u>: the beta coefficient of an intermediate variable in a causal direction remains relatively constant as other system variables are introduced and controlled in stepped regression, whereas that in the acausal direction changes noticeably.

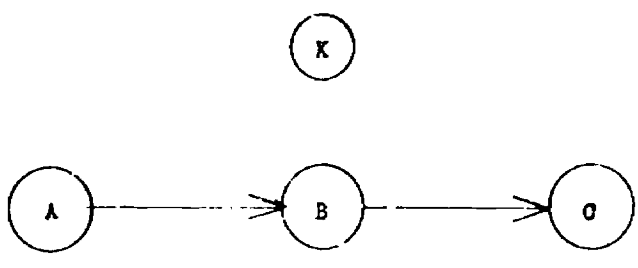

Figure 1. Causal Direction A-B-C

If variables A, B, and C are related causally as in Figure 1 above, and if other system variables K, as appropriate, are controlled, then condition 1 (Inequality 1) of the theorem is exemplified.

From the second of the two regression equations used for each beta differential, the beta coefficient for the end variable, A in predicting variable C or C in predicting variable A, is taken for comparison. Again, if the variables are related causally as in Figure 1 above, condition 2 of the theorem is exemplified.

Correlation comparisons of conditions 3 and 4 are also exemplified in Figure 1 above for variables causally related as shown.

The data for X_i from Equations 5 through 9 relate causally as shown in Figure 2 below. The b weights were set equal to 1 to simplify manipulation and analysis, as in the previous study. The X_i then served as input data in regression, the beta and correlation coefficients as output for analysis.

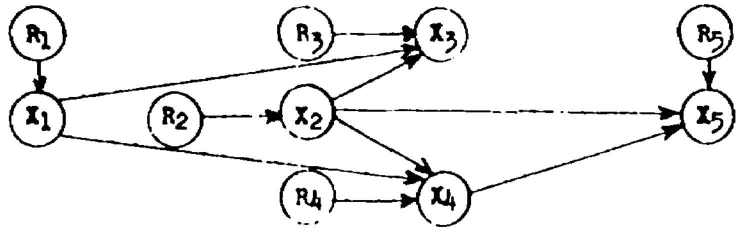

Figure 2. Causal Model of Five Variables X_i

In the initial stages of data processing, certain checks were conducted: (1) the 5x5 R_1 correlation matrix revealed no r significantly different from zero at the .01 level; (2) the left and right members of Equations 5 through 9 were found equal, to five significant figures, when appropriate mean R_1 and X_1 were substituted; and (3) the b weights found in regression equations approximated the b weights in Equations 5 through 9, predicted X_1 and mean X_1 being found equal, to five significant figures.

Following the rules outlined by Blalock[2] final check was made to confirm the efficacy of the model given in Equations 5 through 9 and in Figure 2, as outlined in Table 1 below. Briefly, a "causal model" includes a finite set of explicitly defined variables assumed to be interrelated causally along logically ascertained paths; and it is also assumed that outside variables, while operating, do not disturb the causal patterning within the system. Because the system was actually designed to be causal as shown in Figure 2, the assumptions are not necessary in this model.

Table 1. Correlation Checks for Model from Figure 2[a]

Prediction Equation	Partial Control	Partial Value[c]	Actual Value	Expected Path	Expected Value[d]
$r_{12} = 0$	b	b	-0163	b	b
$r_{15} = 0$	$r_{15\cdot24}$	0323	3750	$r_{14}r_{45}$	4913
$r_{34} = 0$	$r_{34\cdot125}$	0527	6839	$r_{13}r_{14} + r_{23}r_{24}$	6615
$r_{35} = 0$	$r_{35\cdot12}$	-0035	6469	$r_{23}r_{25} + r_{23}r_{24}r_{45}$	7081

Note--Values are checked in two ways, first by the partial showing a true correlation of zero and second by the expected value of correlation products in paths where variables are to be controlled.

a Decimal points precede values listed.
b Check is not necessary by design.
c Value of partial approximates value of prediction equation.
d Value of expected path approximates actual (zero-order) value.
* At df of 200, $r_{.01}$ is greater than or equal to .181[3].

The correlation products in the expected paths, such as in Table 1, are reflected in numerators of the formulas; that is, the second term in the numerator. As such, if appropriate paths are controlled, the sum of such products should approximate the actual (zero-order) value of the correlation and therefore, when subtracted from the actual value, would produce the zero value necessary. In other words, the actual values may be considered spurious if not zero by the designed model. One should check partial r math formulas.

With the final checks of correlations within the model satisfied, the model was initially disregarded. With the five variables, 30 possible three-variable bi-directional paths could be examined; that is, 60 possible directions. However, from Figure 2, disregarding arrowhead leaders, only nine paths need be analyzed: 145, 132, 142, 245, 254, 314, 324, 325, and 425. (Path 145 may also be referred to as path 541; a direction, moreover, may only be referred to only in the direction of the ordered numerals for variables, for example, direction 1-4-5 indicating initial-cause, intermediate, and final-effect variables, in order.) In the

analysis, however, paths 145 and 245 were studied first, since they were the only two designed as causal. For ease of understanding, the correlation matrix of all zero orders and partials was studied first, by triads, in blocks of correlation values for comparisons, for conditions 3 and 4 of the theorem, for each step in regression (partialling); all steps had to yield results consistently the same in order for asymmetry. Betas and beta differentials, procedure outlined above, were then studied, step by step; again, all steps had to yield results consistently the same in order for asymmetry. Any inconsistencies tentatively would indicate dropping a path completely. However, correlations would have to be reanalyzed logically in that conditions 3 and 4 are compounds of three inequalities.

The total variances in predicted variables 3, 4, and 5 were then checked for contribution by component X_i in the standardized regression equations. It was expected that the more dependent an X_i was on system variables, the higher would be the variance accounted for.

DATA SOURCES

Dummy data were generated via programs for IBM 360/40 at the Boston College Computer Center, under the Title IV program, ESEA, PL 89-10 (directed by Dr. John J. Walsh). Random R_i, as described above, were substituted into Equations 5 through 9 to produce X_i, the latter then subjected to stepped-regression and correlation analysis for the model in Figure 2 (originally suggested by Dr. Ronald L. Nuttall, also of Boston College).

RESULTS AND CONCLUSIONS

Blocks of correlations were compared by triads, the first set being that for variables 1, 4, and 5, as presented in Table 2 below. Since there was no link designed for connecting variables 1 and 5, the only two directions to check were 1-4-5 and 5-4-1, and then by comparing these two directly.

Table 2. Correlation Comparisons for Variables 1, 4, and 5, with Controls[a]

Controls[b]	$r_{15 \cdot}$	$r_{14 \cdot}$	$r_{45 \cdot}$	Direction[c]
		For Condition 3[d]		
-	3750	5564	8830	1-4-5
2	5946	7124	8163	1-4-5
3	-0133	2574	7919	1-4-5
23	4592	5629	7699	1-4-5
		For Condition 4[e]		
x	-2981	5177	8754	1-4-5
x2	0323	4888	6959	1-4-5
x3	-3679	4387	8231	1-4-5
x23	0490	3694	6965	1-4-5

a Decimal points precede values listed. b For partials of rs.
c From comparison of absolute values. d Control only for variables X.
e Control for third variable in path as well as for variables X; hence, x used.

In the traditional sense, it seems ridiculous to control variables where it is not logical. For example, holding variable 1 constant should not be done, since variation in variable 4 is reduced by variable 1; hence, r_{45} is reduced. It would also be absurd to hold variable 4, the intervening variable, constant, since variable 5 varies with variable 4 and not directly with variable 1. Nevertheless, the information is easily available with very little extra time or effort on the computer. Real life systems of variables do not seem so a priori as one may hypothesize. And nevertheless, the investigator has chosen to disregard such sound advice for the sake of discovery!

Direction 1-4-5 is the tentative decision for conditions 3 and 4 from the correlation study.

Regression equations in standard form were next studied for path 145, as presented in Table 3 below.

Table 3. Beta Studies for Path 145[a]

	Direction 1-4-5[b]			Direction 5-4-1[c]			Direction[d]	
Other X_i[e]	X_1	X_4	$d\beta_{54}$	$d\beta_{14}$	X_4	X_5	β	$d\beta$
-	-	8830			5564	-		
-	-1684	9767	0937	4659	1.0223	-5277	1-4-5	1-4-5
2	-	6690			8959	-		
2	0173	6535	0155	0403	8556	0602	1-4-5	1-4-5
3	-	8277			2842	-		
3	-2201	8902	0625	5091	7933	-6150	1-4-5	1-4-5
2, 3	-	6730			5804	-		
2, 3	0323	6544	0186	0504	5300	0748	1-4-5	1-4-5

a Decimal points precede values listed except where actually shown.
b Variable 5 is predicted. c Variable 1 is predicted.
d From comparison of absolute values. e Variables in regression equations.

Again direction 1-4-5 is the decision; conditions 1 and 2 are satisfied.

Returning to the model of Figure 2 and then to the correlation matrix for a logical analysis, it appears that to satisfy the system best,

$$|r_{15\cdot24}| < |r_{14\cdot2}| < |r_{45\cdot2}| \,. \qquad (10)$$

The values, respectively, for the members in Inequality 10 are .0323, .7124, and .8163, the order being the same as required by Inequality 10; that is, ascending. Hence, direction 1-4-5 is accepted as causal.

Procedures as in Tables 2 and 3 were repeated for relations among variables 2, 4, and 5. As one may guess, finding causal relations for variables 1, 4, and 5 was fairly simple, because there is no link between variables 1 and 5, as seen in Figure 2. Obviously, path 245 would be difficult to decide the direction of causality, since all three variables are linked, and it was precisely here that a modification to the theorem presented itself, as taken up later. The procedure for a duplication of that in Table 2 produces direction 2-5-4 as the tentative

decision instead of the designed direction 2-4-5; that is, the ascending order of rs is r_{24}, r_{25}, and r_{45}. Despite this deviation from what was expected, it is most important to note that all correlations when compared yielded results consistently the same, for all entries in the blocks. Since it was noted that r_{24} was less than r_{45}, however, the second and third members of conditions 3 and 4 of the theorem, as applied to direction 2-4-5, were met; this would mean a revision to the theorem would be needed to drop the first member or to have same reflected in a new condition. In light of a final step of logic at the macroscopic level, as in Inequality 10, such an added condition along with revision of conditions 3 and 4 seems indicative. (See Table A for correlations.)

The procedure of Table 3 for betas and beta differentials was also repeated for variables 2, 4, and 5. Noting that with an impending revision of conditions 3 and 4 three different directions were feasible, 2-5-4, 2-4-5, and 4-2-5, conditions 1 and 2 were checked in sets of regression equations for each of these. These were: (1) predicting variable 4 from variable 5, then stepping end variable 2 into regression; (2) predicting variable 5 from variable 4, then stepping variable 2 in; and (3) predicting variable 5 from variable 2, then stepping variable 4 in. For comparison, as in the right side of Table 3, the reverses of these directions were compiled. In all three cases, conditions 1 and 2 were met. Again not discouraged, the investigator noted that all steps yielded consistent results. The variables 2, 4, and 5 are so confounded that the decision as to the correct direction, 2-5-4, 2-4-5, or 4-2-5, would have to depend on logic in the final analysis. (No wonder the word "confounded" has an emotional stigma attached to it not unlike Anglosaxonese!) (See Table B for betas.)

Returning to the model of Figure 2 and then to the correlation matrix for a logical analysis, it appears that to satisfy the system best, three inequalities would have to be true; but only one can be true:

For direction 2-5-4, $|r_{24\cdot1}| < |r_{25\cdot4}| < |r_{45\cdot2}|$. (11)

For direction 2-4-5, $|r_{25\cdot4}| < |r_{24\cdot1}| < |r_{45\cdot2}|$. (12)

For direction 4-2-5, $|r_{45\cdot2}| < |r_{24\cdot1}| < |r_{25\cdot4}|$. (13)

The values for the members in Inequality 11 are, respectively and in the same order, 7410, 5974, and 8163; Inequality 12, 5974, 7410, and 8163; and Inequality 13, 8163, 7410, and 5974. Thus, direction 2-4-5 is accepted as causal, in that for Inequality 12 yields values in ascending order and the others do not. (This sounds pragmatic, and it is, but it is considered less so than groping in the dark from personal bias.)

The remaining five paths, 132, 142, 314, 324, and 325, were rejected as causal in that comparisons did not yield consistent results for the four conditions of the theorem, as expected.

A revisit to the previous study[1] revealed that the same results were shown in both studies; that is, for both horizontal and normal distributions of R_1. Exactly what happened to the X_1 could only be guessed at first. In the first study, all distributions were bell-shaped, both R_1 and X_1. But in the present study, X_1 and X_2 were rectangular since they were equated to and determined by R_1 and R_2, respectively; whereas X_3, X_4, and X_5 , when grouped, were found to

be bell-shaped, as indicated in Figure 3 below. Like rolling a number of dice, the probability of getting central values is greater than that of getting extreme values, low or high.

Inequality 7:

Inequality 8:

Inequality 9:

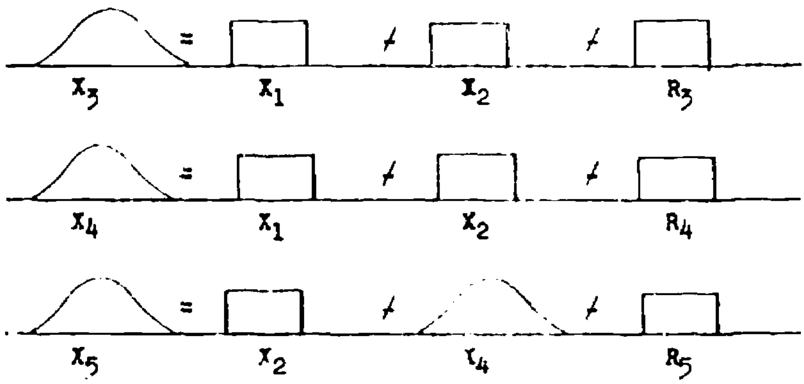

Figure 3. Distributions of Endogenous System Variables

Total variances for predicted variables 3, 4, and 5 were accounted for by both [illegible] and X_i in the previous study to the extent of 100%, as expected. In the present study, the same would be found; hence, total variances were only checked for contributions by X_i, as shown in Table 4 below, such that

$$V_{m\cdot wx} = \beta_{mw\cdot x}\, r_{mw} + \beta_{mx\cdot w}\, r_{mx} \quad . \tag{14}$$

The partitioning shows the amount of variance in the predicted variable contributed by each of the independent variables, X_i, from Equations 7, 8, and 9. The R_i could be taken as "random noise" from outside the system; this is tantamount to the assumption that outside variables, while operating, do not disturb the patterning of variables within the system. (See Table O for total variances.)

Table 4. Variances Accounted For by X_i[a]

Predicted Variance[b]	Variances from Partitions[c]			Total from Partitions[d]
$V_{3\cdot 12}$ 6609	$\beta_{31\cdot 2}r_{13}$ 3562	$\beta_{32\cdot 1}r_{23}$ 3047	- -	6609
$V_{4\cdot 12}$ 6887	$\beta_{41\cdot 2}r_{14}$ 3151	$\beta_{42\cdot 1}r_{24}$ 3746	- -	6897
$V_{5\cdot 24}$ 8583	$\beta_{52\cdot 4}r_{25}$ 2676	$\beta_{54\cdot 2}r_{45}$ 5907	- -	8583
$V_{5\cdot 124}$ 8584	$\beta_{51\cdot 24}r_{15}$ 0065	$\beta_{52\cdot 14}r_{25}$ 2730	$\beta_{54\cdot 12}r_{45}$ 5770	8585

a Decimal points precede values listed.
b From Table O.
c Products of two values shown.
d Sums from this table.

From Table 4 it is noted that the predicted variance and the totals from partitions are equal within rounding error. It is also noted that variable 5, which is "deeper" into the system (see Figure 2) than variables 3 and 4, has more variance accounted for from within the system than variables from outside; except for R_5, variable is further removed from R_1, the outside variables, and determined more from variation within the system. Indirectly, variable 1 contributes to variable 5 and is causally related through variable 4; it could therefore be argued that variable 1 should be in the regression equation for variable 5, but notice how little it contributes to the variance in variable 5.

As a side study of the variance picture, by tradition again, it makes sense to account for as much variance as possible in predicted variables from other variables related to them and stepped into regression to increase the variance. For those who seemingly disregard the actual causal patterning within a system, it should be quite obvious that variable 1 can be predicted from sequentially posterior variables in such a way as to obtain as much as 67% of its variance. Also notice how variable 4 can have as much as 84% of the variance accounted for, if variable 5, a causally posterior variable is stepped in after variables 1 and 2, its direct causal variables, a presumptuous fabrication to the extent of 13% more variance accounted for; so much for the variance accountant!

The problem of using a theorem with four conditions to confirm causal directions for three-variable paths in a closed system of five variables was investigated. It was consistent, both mathematically and logically, for clearcut causal directions, such as direction 1-4-5. It was consistent in part, mathematically, for three-variable paths where variables were confounded, such as path 245, but salvaged by the logic of deciding which variables K to control in computing partials. Clearly, the theorem may be left as is if the stipulation is made to return to conditions 3 and 4 in reference to the first of the three members of the condition; or it would be better to drop the first member of conditions 3 and 4 and add a fifth condition such that

$$|r_{ac\cdot k'}| < |r_{ab\cdot k''}| < |r_{bc\cdot k'''}| \quad , \tag{15}$$

where variables $K^{()}$ follow from logical analysis. Such revision remains for further study. The theorem was also valid for rejecting possible imitators of causal directions. With the one modification on the use of logic after the mathematical analysis, causal directions 1-4-5 and 2-4-5, the only two designed, were not eliminated whereas all others were rejected. The theorem seems to work with horizontal as well as with normal distributions of R_1 in generating a system of X_1, the present study producing the same results of system relations as the previous study. The variance picture for predicted variables was as expected, with attention called to false variance accounting.

From the theorem, one should note that there is a set of consistently logical and mathematical inequalities emulating a family-like relationship. In time, the relationships among a grandfather, father, and son seem logically ordered in ascending order of relational magnitude as grandfather-son, grandfather-father, and father-son. If a characteristic of the son were to be predicted, again it would seem logical to predict from the same characteristic of the father; and if that characteristic of the grandfather were stepped into regression, it should add less to the predicted variance of the son's characteristic than that of the father adds, while the latter should remain relatively stable. If the

same characteristic were to be predicted in the grandfather from the father and that of the son were stepped in, it seems many a son two generations removed has contributed to the old man's unpredictability, no matter how stable in life he is. When other system variables are considered, such as characteristics of wives, siblings, intruders, peers, and other amorphous impedimenta, the father-son relationship seems to remain strong. The theorem poses a mathematical analog as well as a logical one in systematic causal analysis.

The basic problem of the theorem presently is that it may have to be confined to the computer. However, it should at least be tested with real life data, to cross the bridge from theory to practice. Moreover, the overall finding about the theorem is that it is reliable for confirming hypothesized causal directions of three-variable paths and is potentially useful by the practitioner. It is further speculated that the structure of analysis epitomized in Tables 2 and 3 can serve as a basis for determining as well as confirming causal directions; that is, for also generating hypotheses.

SCIENTIFIC OR EDUCATIONAL IMPORTANCE OF THE STUDY

The educational world is largely a nonexperimental and correlational one. Masses of data are available and must be handled. High-speed computers are also available. Yet how frustrating it is to both practitioner and researcher to try to isolate variables to study their relationships! How often does one do with a system of variables with much to be desired, palliating only public relations aspects, settling for something much less than the best! Obversely, how often does one look disdainfully or confused at sophisticated multivariate statistics! Perhaps a multivariate tool such as the theorem in this paper could be the help needed to cross bridges of understanding between survey research and experimental research, theorist and practitioner, researcher and adminstrator, evaluator and researcher. Hypotheses or not, it would make sense or even provide security if guess work were cut down in a measured way, perhaps with the theorem as presented herein. It also seems that such theorem could also cut time in analysis and interpretation, were a computer program to come as a byproduct of the future studies. Indeed, the mathematics alone of the theorem seems to lend to science. If demonstrated with real data, the theorem could abet educational practice while also it could be a door opener for causal theory. Once causal links are established, one may then be able to design an experiment to test some findings from survey research, if feasible. Science and practice have no real borders. Hence, it is expected that follow-up studies in applying the theorem should yield results for both educational theory and practice. But caveat.

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References

1. Nigro, George A., "A Beta Index to Confirm Causal Directions in a Closed System of Five Variables," a paper presented at AERA Annual Meeting, Feb. 1969; also, ED 031 096, ERIC.

2. Blalock, Hubert M., Causal Inferences in Nonexperimental Research, University of North Carolina Press, Chapel Hill, N.C., 1964.

3. Arkin, Herbert, and Raymond R. Colton, Tables for Statisticians, 2nd ed., Barnes and Noble, Inc., N.Y., 1965.

Table A. Correlation Matrix of X_1[a]

Variable X_1	C^b	X_2	C^b	X_3	C^b	X_4	C^b	X_5
1	-	-0163	-	5923	-	5564	-	3750
	3	-5047	2	7184	2	7124	2	5946
	4	-5356	4	3494	3	2574	3	-0133
	5	-4979	5	4947	5	5177	4	-2981
	34	-6744	24	5735	23	5629	23	4592
	35	-6417	25	6398	25	4884	24	0323
	45	-4670	45	4085	35	4387	34	-3679
	345	-6091	245	5743	235	3694	234	0490
2			-	5471	-	6066	-	7586
			1	6911	1	7410	1	8250
			4	2280	3	3805	3	6339
			5	1135	5	-2067	4	5974
			14	5247	13	6119	13	7266
			15	4775	15	0688	14	5431
			45	1923	35	-2571	34	5889
			145	4746	135	0355	134	4963
3					-	6839	-	6469
					1	5293	1	5687
					2	5290	2	4252
					5	3149	4	1256
					12	0354	12	-0035
					15	0791	14	2569
					25	3481	24	-0136
					125	0527	124	-0393
4							-	8830
							1	8754
							2	8163
							3	7919
							12	6959
							13	8231
							23	7699
							123	6965

a Decimal points precede values listed.
b The numbers are those of variables controlled; "-" for zero-orders.
* At df of 200, r_{01} is greater than or equal to .181[3].

Table B. Beta Coefficients of Predictor X_i[a]

Predicted	Predictor X_i									
X_i	c^b	X_1	c^b	X_2	c^b	X_3	c^b	X_4	c^b	X_5
1	-	-	-	-0163	-	5923	-	5564	-	3750
			3	-4858	2	8581	2	8959	2	9123
			4	-5597	4	3980	3	2842	3	-0141
			5	-7084	5	6014	5	1.0223	4	-5277
			34	-6783	24	5665	23	5084	23	5416
			35	-7985	25	6789	25	8556	24	0002
			45	-5810	45	4477	35	7933	34	-6150
			345	-7048	245	5670	235	5300	234	0748
2	-	-0163	-	-	-	5471	-	6066	-	7586
	3	-5244			1	8577	1	8917	1	8898
	4	-5124			4	2485	3	4366	3	6958
	5	-3500			5	0970	5	-2869	4	1.0119
	34	-6705			14	5153	13	6272	13	6886
	35	-5157			15	4071	15	0969	14	8138
	45	-3754			45	1694	35	-3735	34	9788
	345	-5264			145	4050	135	0440	134	6551
3	-	5923	-	5471	-	-	-	6839	-	6469
	2	6014	1	5569			1	5132	1	4943
	4	3068	4	2092			2	5570	2	5461
	5	4070	5	1328			5	5116	4	1952
	24	5805	14	5342			12	0369	12	-0039
	25	6029	15	5599			15	1305	14	3919
	45	3728	45	2183			25	5742	24	-0257
	245	5815	145	5562			125	0766	124	-0607
4	-	5564	-	6066	-	6839	-	-	-	8830
	2	5664	1	6150	1	5459			1	7847
	3	2330	3	3317	2	5025			2	9959
	5	2621	5	-1489	5	1939			3	7576
	23	5460	13	5969	12	0339			12	7411
	25	2793	15	0489	15	0479			13	7610
	35	2426	35	-1769	25	2110			23	8807
	235	2574	135	0286	125	0363			123	7413
5	-	3750	-	7586	-	6469	-	8830	-	-
	2	3875	1	7649	1	6544	1	9767		
	3	-0126	3	5775	2	3310	2	6690		
	4	-1684	4	3528	4	0808	3	8277		
	23	3894	13	7666	12	-0032	12	6535		
	24	0173	14	3625	14	1684	13	8902		
	34	-2201	34	3543	24	-0072	23	6730		
	234	0321	134	3760	124	-0254	123	6544		

a Decimal points precede values listed except where actually shown.

b The numbers are those of variables controlled; "-" for zero-orders.

Table C. Multiple Correlations and Total Variances Accounted for Predicted X_i[a] (Correlation/Variance)

Predictor X_i[b]	Predicted X_i[c] X_1	X_2	X_3	X_4	X_5
1	- /-	0163/0003	5923/3509	5564/3096	3750/1406
12		- /-	8130/6609	8299/6387	8518/7255
123			- /-	8301/6891	8518/7255
124			8132/6614	- /-	9265/8584
125			8130/6609	9162/8395	- /-
1234			- /-	- /-	9266/8587
1235				9165/8399	- /-
1245			8136/6619	- /-	
13		6912/4778	- /-	7092/5030	6470/4186
134		8206/6733		- /-	9014/8125
135		8680/7535		9164/8397	- /-
1345		8662/7538		- /-	
14		7411/5492	7299/5327		8940/7992
145		8259/6822	7507/5636		- /-
15		8250/6807	7469/5608	9158/8387	
2	0163/0003	- /-	5471/2993	6066/3679	7586/5754
23	7185/5162		- /-	7381/5448	8076/6521
234	8182/6695			- /-	9265/8583
235	7863/6182			9026/8146	- /-
2345	8187/6703			- /-	
24	7125/5076		7039/4954		9264/8583
245	7128/5081		7039/4955		- /-
25	5947/3537		6527/4260	8883/7891	
3	5923/3509	5471/2993	- /-	6839/4678	6469/4185
34	6276/3969	6331/4008		- /-	8849/7831
345	6899/4759	7801/6086			- /-
35	5924/3510	7621/5809		8953/8015	
4	5564/3096	6066/3679	6839/4678	- /-	8830/7796
45	6090/3709	7704/5935	6900/4762		- /-
5	3750/1406	7584/5754	6469/4185	8830/7796	

a Decimal points precede values listed. True variances are underlined.

b The numbers are those of variables contributing to variance of predicted variable; that is, "independent" variables in regression equations.

c X_i listed are "dependent" variables in regression equations.

• At df of 200, r_{01} is greater than or equal to .181, .212, .234, or .253 for 2, 3, 4, or 5 variables, respectively[3].